

$$\cos \theta = \Re(e^{i\theta}) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin \theta = \Im(e^{i\theta}) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\sin' = \cos \quad \cos' = -\sin \quad \tan' = \frac{1}{\cos^2} = 1 + \tan^2$$

$$\cos^2 + \sin^2 = 1 \quad e^{i\theta} = \cos \theta + i \sin \theta \quad |\sin x| \leq |x|$$

$$\frac{\sin x}{x} \xrightarrow{x \rightarrow 0} 1 \quad \frac{\tan x}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\cos x = \cos y \iff x \equiv y [2\pi] \text{ ou } x \equiv -y [2\pi]$$

$$\sin x = \sin y \iff x \equiv y [2\pi] \text{ ou } x \equiv \pi - y [2\pi]$$

$$\tan x = \tan y \iff x \equiv y [\pi]$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \quad \tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \cos^2 x - \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos a \cos b = \frac{1}{2} (\cos(a+b) + \cos(a-b))$$

$$\sin a \cos b = \frac{1}{2} (\sin(a+b) + \sin(a-b))$$

$$\sin a \sin b = \frac{1}{2} (\cos(a-b) - \cos(a+b))$$

$$\cos p + \cos q = 2 \cos\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$$

$$\sin p + \sin q = 2 \sin\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$$

$$\cos p - \cos q = -2 \sin\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$$

$$\sin p - \sin q = 2 \sin\left(\frac{p-q}{2}\right) \cos\left(\frac{p+q}{2}\right)$$

On pose  $t = \tan \frac{x}{2}$ , alors

$$\cos x = \frac{1-t^2}{1+t^2} \quad \sin x = \frac{2t}{1+t^2} \quad \tan x = \frac{2t}{1-t^2}$$

$$x^2 + y^2 = 1 \implies \exists \theta \in \mathbb{R}, \quad x = \cos \theta \text{ et } y = \sin \theta$$

$$\exists \varphi, \psi \in \mathbb{R}, \quad \forall \theta \in \mathbb{R},$$

$$a \cos \theta + b \sin \theta = \sqrt{a^2 + b^2} \cos(\theta + \varphi) = \sqrt{a^2 + b^2} \sin(\theta + \psi)$$

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}$$

$$\operatorname{sh}' = \operatorname{ch} \quad \operatorname{ch}' = \operatorname{sh} \quad \operatorname{th}' = \frac{1}{\operatorname{ch}^2} = 1 - \operatorname{th}^2$$

$$\operatorname{ch}^2 - \operatorname{sh}^2 = 1 \quad \exp = \operatorname{ch} + \operatorname{sh}$$

$$\frac{\operatorname{sh} x}{x} \xrightarrow{x \rightarrow 0} 1 \quad \frac{\operatorname{th} x}{x} \xrightarrow{x \rightarrow 0} 1$$

$$\operatorname{ch} x = \operatorname{ch} y \iff x = y \text{ ou } x = -y$$

$$\operatorname{sh} x = \operatorname{sh} y \iff x = y$$

$$\operatorname{th} x = \operatorname{th} y \iff x = y$$

$$\operatorname{ch}(a+b) = \operatorname{ch} a \operatorname{ch} b + \operatorname{sh} a \operatorname{sh} b$$

$$\operatorname{sh}(a+b) = \operatorname{sh} a \operatorname{ch} b + \operatorname{sh} b \operatorname{ch} a$$

$$\operatorname{ch}(a-b) = \operatorname{ch} a \operatorname{ch} b - \operatorname{sh} a \operatorname{sh} b$$

$$\operatorname{sh}(a-b) = \operatorname{sh} a \operatorname{ch} b - \operatorname{sh} b \operatorname{ch} a$$

$$\operatorname{th}(a+b) = \frac{\operatorname{th} a + \operatorname{th} b}{1 + \operatorname{th} a \operatorname{th} b} \quad \operatorname{th}(a-b) = \frac{\operatorname{th} a - \operatorname{th} b}{1 - \operatorname{th} a \operatorname{th} b}$$

$$\operatorname{ch} 2x = 2 \operatorname{ch}^2 x - 1 = 1 + 2 \operatorname{sh}^2 x = \operatorname{ch}^2 x + \operatorname{sh}^2 x$$

$$\operatorname{sh} 2x = 2 \operatorname{sh} x \operatorname{ch} x \quad \operatorname{th} 2x = \frac{2 \operatorname{th} x}{1 + \operatorname{th}^2 x}$$

$$\operatorname{ch}^2 x = \frac{1 + \operatorname{ch} 2x}{2} \quad \operatorname{sh}^2 x = \frac{\operatorname{ch} 2x - 1}{2}$$

$$\operatorname{ch} a \operatorname{ch} b = \frac{1}{2} (\operatorname{ch}(a+b) + \operatorname{ch}(a-b))$$

$$\operatorname{sh} a \operatorname{ch} b = \frac{1}{2} (\operatorname{sh}(a+b) + \operatorname{sh}(a-b))$$

$$\operatorname{sh} a \operatorname{sh} b = \frac{1}{2} (\operatorname{ch}(a+b) - \operatorname{ch}(a-b))$$

$$\operatorname{ch} p + \operatorname{ch} q = 2 \operatorname{ch}\left(\frac{p+q}{2}\right) \operatorname{ch}\left(\frac{p-q}{2}\right)$$

$$\operatorname{sh} p + \operatorname{sh} q = 2 \operatorname{sh}\left(\frac{p+q}{2}\right) \operatorname{ch}\left(\frac{p-q}{2}\right)$$

$$\operatorname{ch} p - \operatorname{ch} q = 2 \operatorname{sh}\left(\frac{p+q}{2}\right) \operatorname{sh}\left(\frac{p-q}{2}\right)$$

$$\operatorname{sh} p - \operatorname{sh} q = 2 \operatorname{sh}\left(\frac{p-q}{2}\right) \operatorname{ch}\left(\frac{p+q}{2}\right)$$

On pose  $t = \operatorname{th} \frac{x}{2}$ , alors

$$\operatorname{ch} x = \frac{1+t^2}{1-t^2} \quad \operatorname{sh} x = \frac{2t}{1-t^2} \quad \operatorname{th} x = \frac{2t}{1+t^2}$$