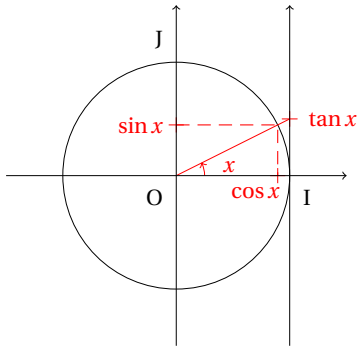
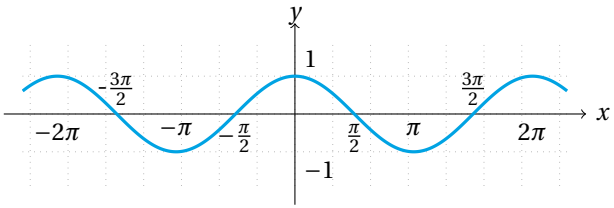
Fonction cos :

2π-périodique, paire, dérivable $\mathbb{R}$.

$$\cos' = -\sin$$





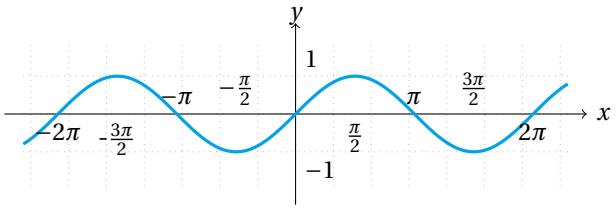
Fonction sin :

2π-périodique, impaire, dérivable sur $\mathbb{R}$.

$$\sin' = \cos$$

$$\frac{\sin x}{x} \xrightarrow[x\to 0]{} 1.$$

$$\forall\, x \in \mathbb{R},\quad |\sin x| \leqslant |x|$$



Fonction tan :

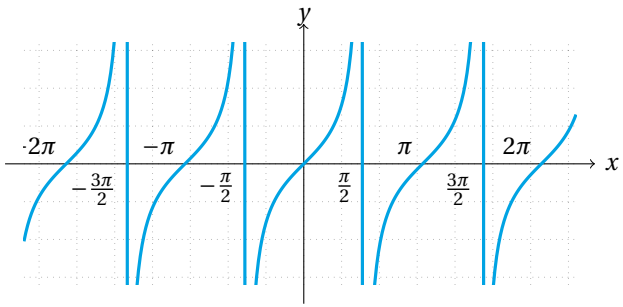
$$\tan : \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\} \to \mathbb{R}$$

$$\tan = \frac{\sin}{\cos}$$

π-périodique, impaire, dérivable sur $\mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$.

$$\tan' = \frac{1}{\cos^2} = 1 + \tan^2$$

$$\frac{\tan x}{x} \xrightarrow[x\to 0]{} 1.$$



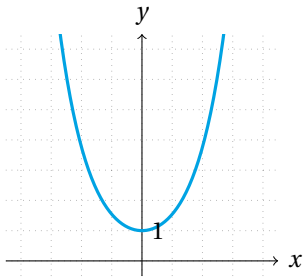
Fonction ch :

$$\mathrm{ch} : \mathbb{R} \to \mathbb{R}$$

$$\mathrm{ch}(x) = \frac{e^x + e^{-x}}{2}$$

Paire, dérivable sur $\mathbb{R}$.

$$\mathrm{ch}' = \mathrm{sh}$$



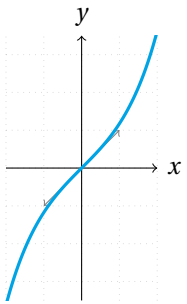
Fonction sh :

$$\mathrm{sh} : \mathbb{R} \to \mathbb{R}$$

$$\mathrm{sh}(x) = \frac{e^x - e^{-x}}{2}$$

Impaire, dérivable sur $\mathbb{R}$.

$$\mathrm{sh}' = \mathrm{ch}$$



Fonction th :

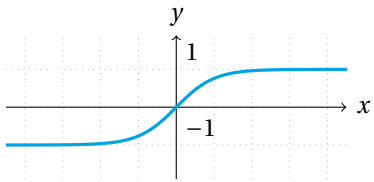
$$\mathrm{th} : \mathbb{R} \to \mathbb{R}$$

$$\mathrm{th} = \frac{\mathrm{sh}}{\mathrm{ch}}$$

Impaire, dérivable sur $\mathbb{R}$.

$$\mathrm{th}' = \frac{1}{\mathrm{ch}^2} = 1 - \mathrm{th}^2$$

$$\mathrm{th}\, x \xrightarrow[x\to\pm\infty]{} \pm 1$$



Fonction Arcsin :

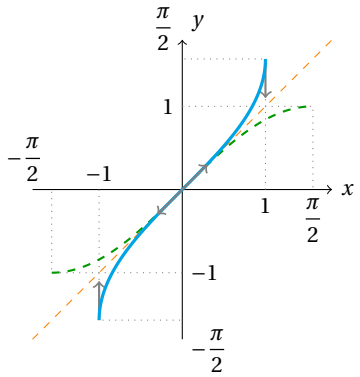
$$\mathrm{Arcsin} = \left(\sin_{\left|\left[-\frac{\pi}{2},\frac{\pi}{2}\right]}\right)^{-1} : [-1,1] \to \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

dérivable sur $]-1,1[$ de dérivée

$$\mathrm{Arcsin}' : x \mapsto \frac{1}{\sqrt{1-x^2}}.$$

$$\mathrm{Arcsin}(\sin x) = x \iff x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

$$\forall x \in [-1,1],\quad \sin(\mathrm{Arcsin}\, x) = x$$



Fonction Arccos :

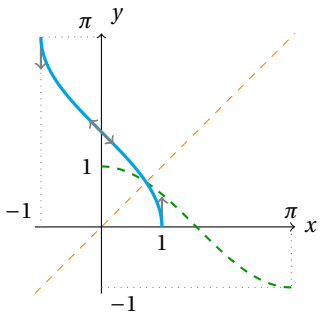
$$\mathrm{Arccos} = \left(\cos_{|[0,\pi]}\right)^{-1} : [-1,1] \to [0,\pi]$$

dérivable sur $]-1,1[$ de dérivée

$$\mathrm{Arccos}' : x \mapsto \frac{-1}{\sqrt{1-x^2}}.$$

$$\mathrm{Arccos}(\cos x) = x \iff x \in [0,\pi]$$

$$\forall x \in [-1,1],\quad \cos(\mathrm{Arccos}\, x) = x$$



$$\forall x \in \mathbb{R},\quad \mathrm{Arccos}\, x + \mathrm{Arcsin}\, x = \frac{\pi}{2}$$

Fonction Arctan :

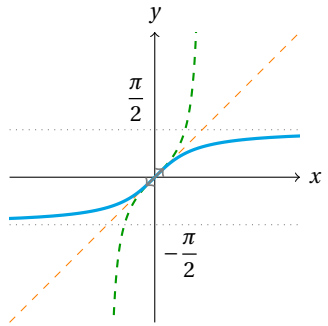
$$\mathrm{Arctan} = \left(\tan_{\left|\left]-\frac{\pi}{2},\frac{\pi}{2}\right[}\right)^{-1} : \mathbb{R} \to \left]-\frac{\pi}{2},\frac{\pi}{2}\right[$$

dérivable sur $\mathbb{R}$ de dérivée

$$\mathrm{Arctan}' : x \mapsto \frac{1}{1+x^2}.$$

$$\mathrm{Arctan}(\tan x) = x \iff x \in \left]-\frac{\pi}{2},\frac{\pi}{2}\right[$$

$$\forall x \in \mathbb{R},\quad \tan(\mathrm{Arctan}\, x) = x$$



$$\forall x \in \mathbb{R}^*,\quad \mathrm{Arctan}\, x + \mathrm{Arctan}\, \frac{1}{x} = \mathrm{sgn}(x)\frac{\pi}{2}$$