Primitive n° 34 : $\frac{\operatorname{ch} x}{1+\operatorname{ch}^2 x}$

Sur $\mathbb{R}$,

On pose $t = \operatorname{sh} x$ $\quad dt = \operatorname{ch} x \, dx$. $\forall t \in \mathbb{R}$, $\operatorname{ch}^2 x = 1 + \operatorname{sh}^2 x$.

Alors, $\displaystyle\int \frac{\operatorname{ch} x}{1+\operatorname{ch}^2 x}\,dx = \int \frac{\operatorname{ch} x}{1+1+\operatorname{sh}^2 x}\,dx = \int \frac{dt}{2+t^2}$.

donc, $\displaystyle\int \frac{\operatorname{ch} x}{1+\operatorname{ch}^2 x}\,dx = \int \frac{dt}{2\left(1+\frac{t^2}{2}\right)} = \frac{1}{2}\int \frac{dt}{1+\left(\frac{t}{\sqrt{2}}\right)^2}$

$$= \frac{1}{\sqrt{2}} \int \frac{\frac{1}{\sqrt{2}}}{1+\left(\frac{t}{\sqrt{2}}\right)^2}\,dt$$

Donc, $\boxed{\displaystyle\int \frac{\operatorname{ch} x}{1+\operatorname{ch}^2 x}\,dx = \frac{1}{\sqrt{2}} \arctan\left(\frac{t}{\sqrt{2}}\right) + C = \frac{1}{\sqrt{2}} \operatorname{Arctan}\left(\frac{\operatorname{sh} x}{\sqrt{2}}\right) + C \text{ avec } C \in \mathbb{R}.}$